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SUGGESTED SOLUTION

SYJC

SUBJECT- MATHEMATICS

& STATISTICS

Test Code – SYJ 6118

BRANCH - () (Date :)

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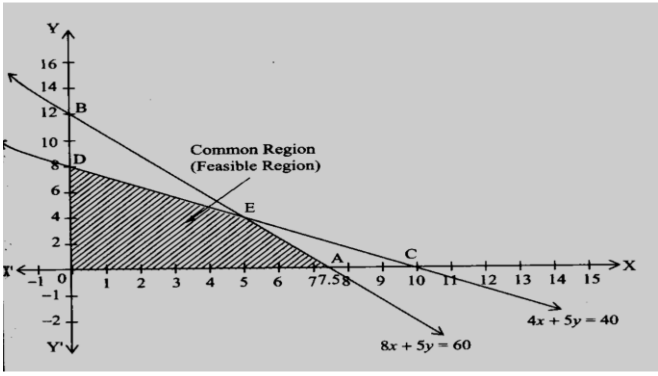
Ans. : 1 **(12)**

1.

Inequation	**Equation**	**Double – Intercept form**	**Points (x, y)**	**Region**
$8x + 5y \leq 60$	$8x + 5y = 60$	$\frac{x}{7.5} + \frac{y}{12} = 1$	A (7.5, 0) B (0, 12)	$8(0) + 5(0) = 0 < 60$ $\therefore$ Origin side
$4x + 5y \leq 40$	$4x + 5y = 40$	$\frac{x}{10} + \frac{y}{8} = 1$	C(10, 0) D(0, 8)	$4(0) + 5(0) = 0 < 40$ $\therefore$ Origin side
$x \geq 0$	$x = 0$	-	-	RHS of y – axis
$y \geq 0$	$y = 0$	-	-	Above x - axis



The shaded portion OAED represents the common region.

(02)

2. Given : $e_3^o = 1.7$, $l_3 = 75$

We know that,

$$e_x^o = \frac{T_x}{l_x}$$

$$\therefore e_3^o = \frac{T_3}{l_3}$$

$\therefore 1.7 = \frac{T_3}{75}$ $\therefore 75 \times 1.7 = T_3$

$\therefore T_3 = 127.5.$

(02)

3. $x - y = 1$, $x = 3$

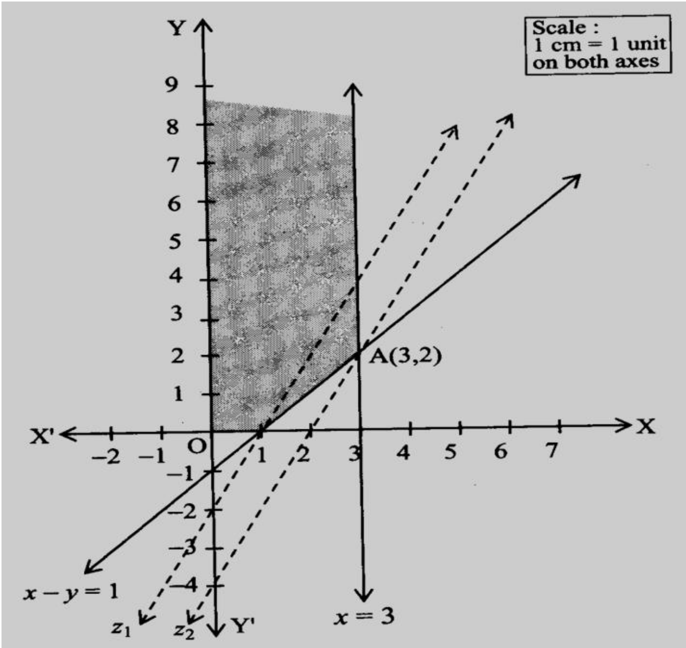
$\frac{x}{1} + \frac{y}{(-1)} = 1$

Let $Z_1 = 2$ $\therefore 2x - y = 2$

$Z_2 = 4$ $\therefore 2x - y = 4$

In this LPP, though feasible region is unbounded, we get unique optimum solution.

$x = 3, y = 2, z = 4$



(02)

4. We present the computation in the following table.

Age group (years)	Population $_nP_x$	No. of Deaths $_nD_x$	Age – SDR per thousand $= \frac{_nD_x}{_nP_x} \times 1000$
0 – 15	12,000	290	24.16
15 – 35	30,000	410	13.66
35 – 70	45,000	495	11.00
70 and above	4,000	168	42.00

(02)

5. $5(x - 7) > -10 \Rightarrow 5x - 35 > -10$

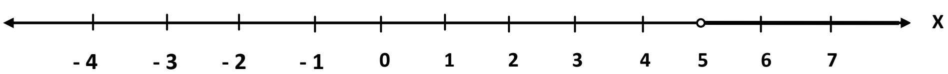
$\Rightarrow 5x > -10 + 35$

$\Rightarrow 5x > 25$

$\Rightarrow x > 5$

$\therefore$ solution interval : $(5, \infty)$

Solution graph : As shown in the figure



(02)

6. Given : Total number of deaths $\Sigma D_i = 900$

$\Sigma P_i = 9000 + 25000 + 32,000 + 9000$

$= 75,000$

Now, CDR $= \frac{\Sigma D_i}{\Sigma P_i} \times 1000$

$= \frac{900}{75000} \times 1000$

$= \frac{900}{75}$

$\therefore$ CDR = 12

Hence, CDR = 12 per thousand.

(02)

Ans.: 2 **(12)**

1. Let x_1 : number of units of Food F_1
and x_2 : number of units of Food F_2

Table

Product	**Food F_1**	**Food F_2**	**Minimum Requirement**
Vitamins	200	100	4000
Minerals	1	2	50
Calories	40	30	1500
Cost / unit	Rs. 50	Rs. 75	

Sick person's problem is to determine x_1 and x_2 so as to minimize the total cost

$Z = 50x_1 + 75x_2$

Subject to constraints

$200x_1 + 100x_2 \geq 4000$

$x_1 + 2x_2 \geq 50$

$40x_1 + 30x_2 \geq 1500$

$x_1 \geq 0, x_2 \geq 0$

(03)

2. Given : $l_4 = 60$, $L_4 = 45$, $p_4 = ?$

We have, $L_x = \frac{l_x + l_{x+1}}{2}$

$\therefore L_4 = \frac{l_4 + l_5}{2}$

$\therefore 45 = \frac{60 + l_5}{2}$

$\therefore 2 \times 45 - 60 = l_5$

$\therefore l_5 = 90 - 60$

$\therefore l_5 = 30$

We have, $d_x = l_x - l_{x+1}$

$\therefore\ d_4 = l_4 - l_5$

$\therefore\ d_4 = 60 - 30 = 30$

We have, $p_x = 1 - q_x$

$\therefore\ p_4 = 1 - q_4$

$= 1 - \frac{d_4}{l_4}$ ....$\left(\because q_x = \frac{d_x}{l_x}\right)$

$= 1 - \frac{30}{60}$

$= 1 - 0.5$

$= 0.5$

Hence, $p_4 = 0.5$.

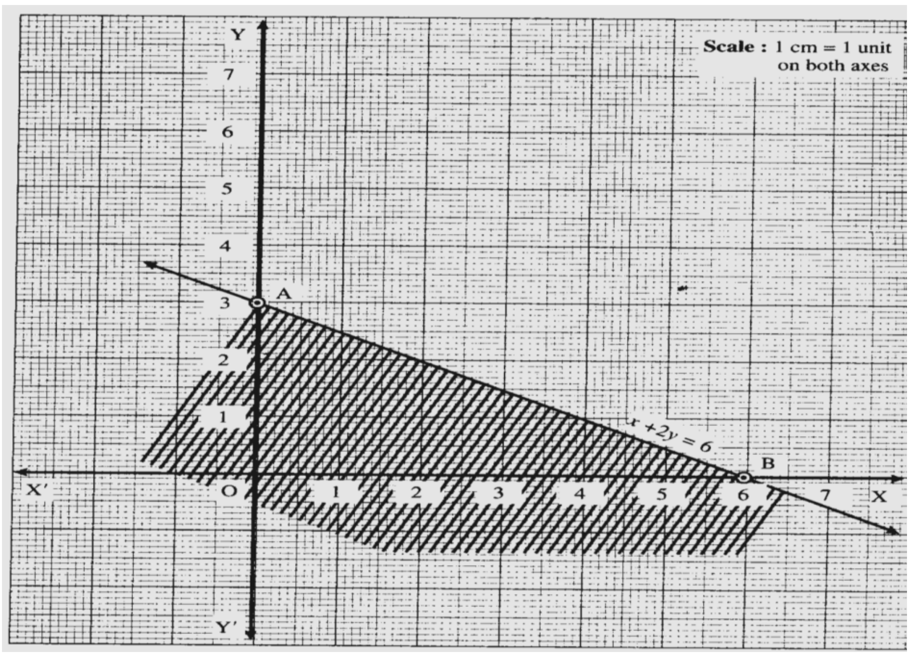
(03)

3. Consider the equation x + 2y = 6.

To draw the graph of this equation, we find two points as follows :

Points	x	y
A	0	3
B	6	0

Two points are A(0, 3) and B(6,0). Draw the graph of this line AB. Choose a point (1,1). The co-ordinate of this point satisfy the given inequations. Therefore, shade the half plane containing this point. The shaded portion as shown in the figure represents the solution graph of the given inequation.



(03)

4. Given : $l_0 = 1{,}000$, $l_1 = 880$, $l_2 = 876$, $T_2 = 3323$

We have, $L_x = \frac{l_x + l_{x+1}}{2}$

$\therefore L_0 = \frac{l_o + l_1}{2} = \frac{1000+880}{2} = \frac{1880}{2} = 940$

$L_1 = \frac{l_1 + l_2}{2} = \frac{880+876}{2} = \frac{1756}{2} = 878$

We have, $T_x = L_x + T_{x+1}$

$\therefore T_1 = L_1 + T_2$

$= 878 + 3323$

$= 4201$

Now, $T_0 = L_0 + T_1$

$= 940 + 4201$

$= 5141$

Now, we calculate e_0^o, e_1^o, e_2^o :

We have, $e_x^o = \frac{T_x}{l_x}$

$\therefore e_0^o = \frac{T_0}{l_0} = \frac{5141}{1000} = 5.141$

$e_1^o = \frac{T_1}{l_1} = \frac{4201}{880} = 4.7738$

$e_2^o = \frac{T_2}{l_2} = \frac{3323}{876} = 3.7933.$

(03)

Ans.: 3 **(16)**

1. Given : $Z = 3x_1 + x_2$

$5x_1 + 9x_2 \le 45$, $x_1 + x_2 \ge 2$, $x_2 \le 4$ x_1, $x_2 = 0$

Now, $5x_1 + 9x_2 = 45$

$\therefore$ two points are (9, 0) (0, 5)

$x_1 + x_2 = 2$

$\therefore$ two points are (2, 0), (0, 2)

$x_2 = 4$

$\therefore$ point is (0, 4).

In graph ABCDE is feasible region. It is a convex polygon whose vertices are A(2,0), B(0, 2), C(0,4), D(1.8, 4), E(9, 0). At at least one of the vertices the value of objective function Z will be minimum.

At A(2, 0), Z = 3(2) + 0 = 6 + 0 = 6

B(0, 2), Z = 3(0) + 2 = 0 + 2 = 2

C (0, 4), Z = 3(0) + 4 = 0 + 4 = 4

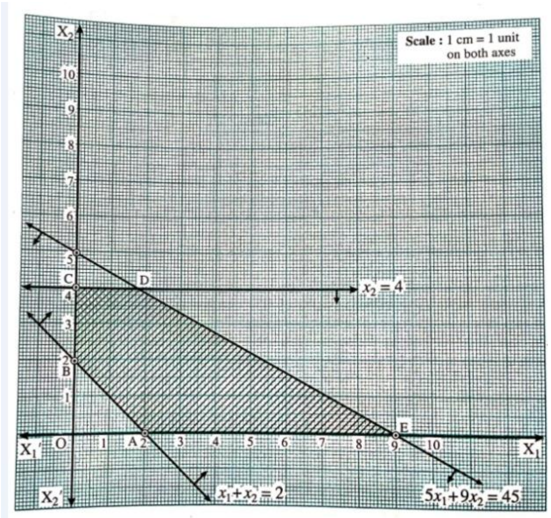
D(1.8, 4), Z = 3(1.8) + 4 = 5.4 + 4 = 9.4

E(9, 0) Z= 3(9) + 0 = 27 + 0 = 27

$\therefore$ at B(0, 2) the value of Z is minimum.

Hence, optimum solution is,

$x_1 = 0$, $x_2 = 2$ and $Z_{min} = 2$.



(04)

2. The calculations are done as follows :

We use, $d_x = l_x - l_{x+1}$

Thus $d_0 = l_0 - l_1$

$= 1000 - 940 = 60$ etc.

Use : $q_x = \frac{d_x}{l_x}$

Thus $q_0 = \frac{d_0}{l_o}$

$= \frac{60}{1000}$

= 0.06, etc.

Use $L_x = \frac{l_x + l_{x+1}}{2}$

Thus, $L_0 = \frac{l_0 + l_1}{2}$

$= \frac{1000 + 940}{2}$ = 970, etc.

The complete life table for the parrots is given below :

Age x	l_x	$d_x = l_x - l_{x+1}$	$q_x = \frac{d_x}{l_x}$	$p_x = 1 - q_x$	$L_x = \frac{l_x+l_{x+1}}{2}$	T_x	$e_x^0 = \frac{T_x}{l_x}$
0	1000	60	0.0600	0.94	970.0	2835.	2.8350
1	940	160	0.1702	0.8298	860.0	1865.0	1.9840
2	780	190	0.2435	0.7565	685.0	1005.0	1.2885
3	590	565	0.9576	0.0424	307.5	320.0	0.5424
4	25	25	1.0000	0	12.5	12.5	0.5000
5	0	-	-	-	-	-	-

(04)

3. Let x = Number of units of chemical A.

y = Number of units of chemical B.

Since, the number of units cannot be negative, $x \geq 0$, $y \geq 0$.

From the given data, we get the following inequations :

$x + 2y \geq 80$

$3x + y \geq 75$

Let Z = Total Cost

The cost of one unit of chemical A is Rs. 4 and that of chemical B is Rs. 6.

$\therefore$ the objective function to be minimized is $Z = 4x + 6y$

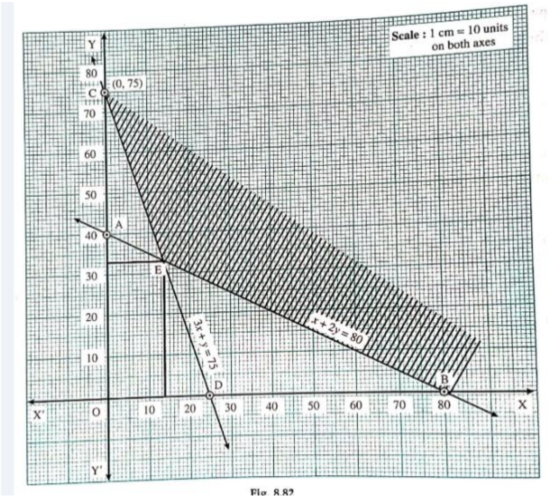
Thus, the LPP is formulated as follows :

Minimize $Z = 4x + 6y$

Subject to $x + 2y \geq 80$, $3x + y \geq 75$, $x \geq 0$, $y \geq 0$.

To draw the graph, we prepare the following table :

Inequation	**Equation**	**Points (x, y)**				**Region**
$x + 2y \geq 80$	$x + 2y = 80$	x	0	80	A(0, 40)	$0 + 2(0) \ngtr 80$
		y	40	0	B(80, 0)	$\therefore$ Non origin side of the line AB
$3x + y \geq 75$	$3x + y = 75$	x	0	25	C(0, 75)	$3(0) + 0 \ngtr 75$
		y	75	0	D (25, 0)	$\therefore$ Non – origin side of the line CD
$0 \geq, y \geq 0$	$x = 0, y = 0$	-				First quadrant



From the graph the unbounded feasible region is CEB. The lower vertices of this region are C(0, 75), E (14, 33), B(80, 0).

At any one of these vertices, the value of Z is minimum.

Z = 4x + 6y

$\therefore$ at C(0, 75), Z = 4(0) + 6(75) = 450

E (14, 33), Z = 4(14) + 6(33) = 56 + 198 = 254

B(80, 0), Z = 4(80) + 6(0) = 320

The value of Z is minimum at the point E (14, 33).

$\therefore$ the solution of the given LPP is x = 14, y = 33, Z_{min} = 254.
Hence, 14 units of chemical A and 33 units of chemical B should be produced so as the cost is minimum.

(04)

4. Given : l_{80} = 717, d_{80} = 214, q_{81} = 0.3364, p_{82} = 0.62006,

l_{81} = ? l_{82} = ?, l_{83} = ?

We have, $d_x = l_x - l_{x+1}$

$\therefore\ d_{80} = l_{80} - l_{81}$

$\therefore\ 214 = 717 - l_{81}$

$\therefore\ l_{81} = 717 - 214$

$\therefore\ l_{81} = 503$

We have, $q_x = \frac{d_x}{l_x}$

$\therefore\ q_{81} = \frac{d_{81}}{l_{81}}$

$\therefore\ 0.3364 = \frac{d_{81}}{503}$

$\therefore d_{81} = 503 \times 0.3364$

$\therefore\ d_{81} = 169.21 \approx 169$

We have, $d_x = l_x - l_{x+1}$

$\therefore\ d_{81} = l_{81} - l_{82}$

$\therefore\ 169 = 503 - l_{82}$

$\therefore\ l_{82} = 503 - 169$

$\therefore\ l_{82} = 334$

We have, $p_x = \frac{l_{x+1}}{l_x}$

$\therefore\ p_{82} = \frac{l_{83}}{l_{82}}$

$\therefore\ 0.62006 = \frac{l_{83}}{334}$

$\therefore\ l_{83} = 334 \times 0.62006$

$\therefore\ l_{83} = 207.1 \approx 207$

Hence, $l_{81} = 503$, $l_{82} = 334$ and $l_{83} = 207$.

(04)